The SoTL Grant Proposal:

EXPLORING THE TEACHING AND LEARNING OF FUNCTIONS

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EXPLORING THE TEACHING AND LEARNING OF FUNCTIONS
Abstract

This project is a semester long teaching-experiment that examines a holistic and developmental approach of the teaching and learning of functions. The study examines the effectiveness of a teaching-learning intervention of the concept of function and families of functions which is nothing else than the actual curriculum of the Pre-calculus course at UNCC. The concept of function provides the foundation for STEM, business, and other majors because of its fundamental significance in any area of knowledge that has to do with mathematics and statistics. Although this concept is part of the mathematics high school curriculum, students arrive to this course with disconnected meanings. If we can improve students' conceptualization of functions, then we accomplish an important mission of our department by preparing the recruitment pool for STEM and other majors for future mathematics courses. The project's teaching-learning intervention includes: (a) scaffolding of the curriculum through tasks that guide inquiry, reading, interpretation, writing, and reflection; it refocuses the traditional course content to emphasize both the concept of function and the invariances of functions across families as well as problem solving in real-world applications; (b) scaffolding students' involvement on their own learning; and (c) constant academic support for each of the four pretest-test-posttest sequences on each family of functions. We will assess the effectiveness of the above teaching-learning intervention of the concept of function and families of functions through a variety of quantitative measures (scores on all four pretest-test-posttest sequence of tests and the common final exam).

Budget Revision Requestes for SOTL Grant (1-8-2019)
Year   2018-2019

Joint Proposal?   _X_ Yes   ___ No

Title of Project   **EXPLORING THE TEACHING AND LEARNING OF FUNCTIONS**

Duration of Project   12 months

Primary Investigator(s)   Adalira Saenz-Ludlow, Anna Athanosopoulou

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UNC Charlotte SOTL Grants Previously Received (please names of project, PIs, and dates)   NA

Allocate operating budget to Department of   Mathematics & Statistics

Account #	Award	Year One January to June
Faculty Stipend	Transferred directly from Academic Affairs to Grantee on May 15	$7700 -
911250	Graduate Student Salaries	$2700
911300	Special Pay (Faculty on UNCC payroll other than Grantee)	
915000	Student Temporary Wages	
915900	Non-student Temporary Wages	
920000	Honorarium (Individual(s) not with UNCC)	
921150	Participant Stipends	
925000	Travel - Domestic	
926000	Travel - Foreign	
928000	Communication and/or Printing	
930000	Supplies (Student Notes Booklets for two classes of 40 stud.)	
942000	Computing Equipment	
944000	Educational Equipment	
951000	Other Current Services	
	Year One Total	$ 10,400

Account #	Award	Year Two July to June
Faculty Stipend	Transferred directly from Academic Affairs to Grantee on May 15	$ -
911250	Graduate Student Salaries	
911300	Special Pay (Faculty on UNCC payroll other than Grantee)	
915000	Student Temporary Wages	
915900	Non-student Temporary Wages	
920000	Honorarium (Individual(s) not with UNCC)	
921150	Participant Stipends	
925000	Travel - Domestic	
926000	Travel - Foreign	
928000	Communication and/or Printing	
930000	Supplies	
942000	Computing Equipment	
944000	Educational Equipment	
951000	Other Current Services	
	Year Two Total	$ -
	GRAND TOTAL (Year One + Year Two)	$ - 10,400

Attachments:

1. Attach/provide a narrative that explains how the funds requested will be used.

2. Has funding for the project been requested from other sources? ___ Yes _X_ No. If yes, list sources.

Budget Narrative

Faculty Stipends ($7700)

A summer stipend of $3850 will be paid to each of the co-PIs as compensation for the approximate 350 hours each will work on the project during Spring, Summer and Fall 2019. The co-PIs have experience conducting studies that include quantitative analysis of student learning. Based on our prior projects, we expect 200 of these hours will be spent analyzing the quantitative data during Spring and Summer 2019; 100 hours in Summer 2019 for preparing detailed written concept case studies; and 50 hours in Fall 2019 to prepare paper proposals and presentations for the dissemination of results. To be able to complete the project, we plan to teach no more than one term during Summer 2019.

Graduate Student Salary ($2700.00)

The graduate teaching assistant will assist the Co-PIs during the Spring semester. She or he will assist in support of given students' feedback on each of the four pretest-test-posttest sequences. The graduate student will also help with tutoring during the weeks that given feedback is not on the agenda. The graduate assistant will work for 10 hours per week for 15 weeks, at a rate of $18.00/hour for a total of $2700.00 during Spring 2019.

UNC CHARLOTTE
College of Liberal Arts & Sciences

Office of the Dean

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October 29, 2018

SOTL Grants Committee
Center for Teaching and Learning
ctl@uncc.edu

Dear Committee Members:

On behalf of Dean Nancy Gutierrez in the College of Liberal Arts & Sciences, I am writing this letter in support of the SOTL proposal submitted by Dr. Adalira Saenz-Ludlow and Dr. Anna Athanasopoulou from the Department of Mathematics & Statistics which is entitled, "Exploring the Teaching and Learning of Functions."   This proposal aligns with the college's and university's focus on student success, retention, and graduation in STEM fields. Although this work focuses on the mathematical concept of functions, success in this concept, and a variety of others, impacts student success in a variety of STEM disciplines. The study proposed here also aligns with the work of the Math Pathways Task Force at the UNC System level. This proposal focuses on curriculum development and activities that will improve student performance in the pre-calculus course, MATH 1103. The outcomes indicated closely align with the university and System-level efforts to investigate the issues that attribute to high DFW rates in mathematics and other STEM courses.

Sincerely,

Banita W. Brown
Associate Dean for Academic and Student Success
College of Liberal Arts and Sciences
Associate Professor of Chemistry

Project Narrative

A. Specific Aims

Goals and Purposes

The proposed study is a semester long teaching-experiment and examines precalculus

students' understanding of the concept of function and families of functions to improve their

conceptualizations in a holistic and developmental approach. It will examine the effectiveness of a

teaching-learning intervention of the concept of function and families of functions which is the

actual curriculum of the precalculus course (MATH 1103) at UNCC. The concept of function

provides the mathematical foundation for STEM, business, and other majors.

We focus on this course for two reasons. First, this course provides the necessary

mathematical foundation for all majors for which mathematics is indispensable and prepares

students for the following mathematics courses. Second, when students develop connected

meanings of the different aspects of the concept of function, they will be able to see the invariances

of such concept across different families of functions. If we improve students' mathematical

preparation, maybe we will increase the retention of students in these mathematics courses. Then,

we would have accomplished an important mission of our department and the university.

The organization of the curriculum and the design of mathematical tasks was done during

the fall semester 2017 and the spring semester 2018. With this organization we establish conceptual

links among what is typically presented as a set of disparate topics loosely connected by means of

applying formal algorithms without expectations of writing complete solutions of problems.

We are mindful of the challenges that exist at the University. The formal class meetings are

lecture-based. Instructors typically teach with the expectation that students will be attentive, ask

good questions, take good notes, and depart with at least a skeletal outline of the lecture that will aid

the completion of course assignments. Also, most instructors do not allow active engagement that is necessary for meaningful learning to occur.

We believe that our organization of the curriculum and the design of the activities combined with the teaching approach and the involvement of the students in reading, writing, and reflecting will improve the students learning experience in mathematics. In two prior courses (fall 2017 and spring 2018) we have seen students improve their knowledge of functions. The added expectation of giving complete written solutions to each of the four pretest-test-posttest sequences and the feedback they will receive for each of them, in addition to the completion of the homework on the Student Notes and online homework, allows for the development of connections among the different aspects of the concept of function. At different points of the students' mathematical careers they have been presented with different definitions of functions in a disconnected manner. In the Student Notes we take into account these different definitions of functions, their properties, their graphs and the operations that are invariant across families of functions. The families considered are piecewise, polynomial, rational, exponential, logarithmic, and trigonometric functions. Application of these functions are taken in physics, chemistry, business, and other fields.

The study will document students' academic performance on objective measures (grades on pretests, tests, posttests, homework, and common final exams). These measures will indicate students' level of understanding of the concept of function and families of functions.

Objectives

This proposal seeks support for Phase 2 of a year-long study. The Phase 1 of the study (three semesters, fall 2017, spring 2018, fall 2018) reorganized and scaffolded the curriculum of the precalculus course and designed sequences of inquiry-base tasks for each family of functions, to build up the Student Notes. This semi-guided inquiry approach demands reading and writing as well thinking and reflection on the part of the students. These Notes enable both the teaching of each

class and the engagement of the students in their own learning while eliciting relational understanding and productive critical thinking.

During the two semesters when drafts of the Notes were used, students performed higher than students in other sessions of MATH 1103 in the common final. In the fall 2017 these students' average was 8 points above the second highest average, and in the spring 2018 the students' average in the common final exam was 18 points above the second highest average of the other sessions. Co-PI Athanasopoulou is using these Student Notes this fall semester, and we are looking forward seeing students' average in the common final.

Co-PI Saenz-Ludlow will teach two sections of MATH 1103 during the spring 2019. We will apply what we learned from Phase 1 to modify the interventions as needed. We will assess the effectiveness of these interventions through a quantitative analysis of scores (data) on all four pretest-test-posttest sequences, and the common final exam to answer each research question.

Research Questions

The study will explore four key questions.

1. Which are necessary algebraic skills that are not sufficiently developed on precalculus students? Why the lack of these skills become perennial stumbling blocks in their conceptualizations of function and operations with them in function families, as measured by written-complete solutions in each one of the four in-course pretest-test-posttest sequences and the common final exam?

2. What is the effect, on precalculus students, of the inquiry-base-designed-activities to facilitate the emergence of connected meanings of the concept of function and operations with them in function families, as measured by scores on the four in-course pretest-test-posttest sequences, and the common final exam?

3. What is the effect, on precalculus students, of the inquiry-base-designed-activities to generate connected meanings for inverse and reciprocal functions, as measured by scores on the four in-course pretest-test-posttest sequences, and the common final exam?

4. What are the indicators of the effect of the pedagogical intervention in this project, as measured by scores on the course final exam?

B. Rationale and Literature Review

To remain a global innovator, the US must be able to provide high quality education to students in the fields of science, technology, engineering, mathematics (STEM) and statistics. It is important to notice that employment in STEM occupations are projected to grow by 8.9 percent from 2014 to 2024, compared to 6.4 percent growth for non- STEM occupations (U.S. Dept of Commerce, 2017). This data suggests that about 2 million additional STEM and other college graduates will be needed over the next decade to fill America's economic demand.

Several important issues must be addressed to meet these needs. In terms of formal academic preparation, we note that the study of functions plays an important role as a source of foundational mathematical knowledge required of every STEM major and of other majors. Students in all areas of knowledge will need to have a strong foundation in mathematics, particularly the study of calculus for which precalculus is a necessary stepping stone (Bressoud, Carlson, Mesa, & Rasmussen, 2013).

Researchers has shown that students hold compartmentalized meanings of the function concept (Brown & Reynolds, 2007; Engelke et al., 2005, Lucus 2005; Vidakovic, 1996, 1997). These researches also argue that students are restricted to carrying out particular analytic techniques and graphical representations with no connections between them. For example, Brown and Reynolds (2007) noticed when students were asked about the inverse functions of a given function they mentioned that they have to switch the domain and range. However, they gave the reciprocal of

the function instead of the inverse function without taking into account domain and range. Engelke et al. (2005), in a study with 1031 students, suggest few students maintained inverse function connected meanings to support them in addressing tasks related to inverse functions correctly.

Gaps in the literature on students' inverse function meanings still remain. Brown and Reynolds (2007) inferences were limited because the students in their small sample (N=7) provided similar responses to tasks. Engelke et al. (2005) has a large sample, but their analyses were limited to reporting students' responses to multiple-choice items. We argue here that one of the reasons students confuse the inverse and reciprocal functions of a given function is because they have disconnected meanings of: operations with functions; multiple representations of functions; and multiple definitions of function. We also argue that students do not internalize those meanings when used algorithmic manner. The Student Notes not only scaffold the tasks but also invite students to integrate those meanings by reading, writing, interpreting, and reflecting. No one can focus attention on all the aspects of the function concept at the same time, but incrementally certain aspects will be integrated, and then others, until awareness of all aspects of this concept is achieved simultaneously (Marton and Booth, 1997).

Impact

The project's interventions will benefit all enrolled students especially those aspiring to major in a STEM field and in other fields that use mathematics. This project will impact 80 students in their progressive understanding of the function concept.

C. Methods

The teaching-experiment methodology (Steffe & Thompson, 2000) is a non-dualistic model of teaching and learning to improve students' learning by eliciting awareness about their own conceptualizations. It approaches knowledge and knowing as actively built up by the individual in idiosyncratic ways (von Glasersfeld, 1995); therefore, researchers can only make inferences about

students' constructed meanings based on their linguistic expressions, and actions like reading, speaking, interpreting, and writing. When making such inferences, researches make second order models of students' ways of operating; this is what Steffe and Thomson (2000) refer to as the *mathematics of students*. This teaching methodology draws from the genetic epistemology of Piaget (1970) and its implied principles of learning, while taking into account the socio-cultural aspects in the construction of meanings (Vygotsky, 1978; Kozulin, 1998).

This teaching-experiment differs from traditional teaching in the following ways: 1. The instructor and the instructional support team work as a research team, attending all classes, and meeting regularly to provide written feedback to students, discuss the progress of the class, and other pedagogical actions for future classes while making changes as necessary; 2. The instructional activities (Student Notes) become the focus of on-going hypothesis testing about the students' learning; 3. The data includes students' written work of classroom assignments, and students' written pretest-test-posttest sequence for each of the four tests. We will use quantitative measures (scores on all semester exams, pretest-test-posttest sequences and the common final exam). The quantitative data will serve to document concept case studies.

D. Evaluation

We look to provide answers to the research questions as follows.

Question 1: We will permanently assess students' prior algebraic skills through written-complete solutions in each one of the four pretest-test-posttest sequences and the common final exam. One of the causes that stumble students' progress is the lack of prior algebraic skills, necessary to understand the concept of function, the properties of functions, and the operations with functions that are invariant across families of functions. Such skills or the lack of them will be continually assessed and quantitatively analyzed.

Question 2: This question will permanently be assessed through written-complete solutions in the four pretest-test-posttest sequences and the common final exam. The same aspects of the concept of function (the characteristics of the graph and behavior of the functional values, operations with functions, and applications) will be the object of study within each family of function. Students' progress or lack of it will be indicated by the grades of each pretest-test-posttest and the common final exam. Students' written-complete solutions will be graded and analyzed. Each sequence of tests gives students the opportunity to make mistakes and to correct them while providing quantitative data for analysis.

Question 3: A function and its inverse and a function and its reciprocal have correlated characteristics. The tasks in the Students Notes are scaffolded to allow students to find similarities and differences between these two functions. Students' progress or lack of it will be demonstrated by the grades of each pretest-test-posttest and the common final exam. Students' written-complete solutions will be graded and analyzed. Each sequence of tests gives students the opportunity to make mistakes and to correct them while providing quantitative data for analysis.

Question 4: Students written-complete-answers to a *common final exam* based solely on multiple-choice items, the computer grade of that exam, the analysis of the complete written answers of each student, and the quantitative comparison of class averages for this course will be indicators of the effect of the pedagogical intervention in this project.

E. Knowledge Dissemination

We plan to present our findings to the University Community through campus teaching and learning outlets including the UNC Charlotte Teaching Week, and the Charlotte Teachers Circle. At the state level, we plan a presentation at the 2019 annual conference of the North Carolina Council of Teachers of Mathematics. At the national level, we plan to submit papers to the annual

conferences of the MAA/AMS Joint Meeting and the international conference for the psychology of mathematics education (PME).

F. Human Subjects

We are preparing the IRB to collect and analyze data in the Spring 2019.

G. Extramural Funding

We will use the results of this study in support of a grant proposal that will be submitted to the Institute of Education Sciences (IES) of the U.S. Department of Education in August 2019, and the Spencer Foundation in November of 2019, institutions that fund projects with freshman and junior college students to influence their learning practices and their capacity to go into science careers and other professions.

H. Timeline

Date	Project Task
November 2018	Submit IRB (Co-PI Athanasopoulou)
January – May 2019	Teach MATH 1103 class (MWF) (Co-PI Saenz-Ludlow and Graduate Assistant) Weekly Meetings of Project Team (Co-PIs, Graduate Assistant)
January 2019	Analysis of first pretest-test 1-posttest/Possible Individual Interviews #1 (Co-PIs Saenz-Ludlow and Athanasopoulou)
February 2019	Analysis of first pretest-test 2-posttest/Possible Individual Interviews #2 (Co-PIs Saenz-Ludlow and Athanasopoulou)
March 2019	Analysis of first pretest-test 3-posttest/Possible Individual Interviews #3 (Co-PIs Saenz-Ludlow and Athanasopoulou)
Abril 2019	Analysis of first pretest-test 4-posttest/Possible Individual Interviews #4 (Co-PIs Saenz-Ludlow and Athanasopoulou)
May 2019	Analysis of Common Final Exams (Co-PIs Saenz-Ludlow and Athanasopoulou)
May-December 2019	Concept Case Studies prepared (Co-PIs Saenz-Ludlow and Athanasopoulou) Report findings via dissemination plan

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